Ideal equal Baire classes

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Joint work with Marcin Staniszewski

Three notions of ideal convergence

Let $\mathcal{I}, \mathcal{J}$ be ideals on ω and let X be a set. Suppose that $(x_n) \subseteq \mathbb{R}$, $x \in \mathbb{R}$, $(f_n) \subseteq \mathbb{R}^X$ and $f \in \mathbb{R}^X$.

- $x_n \xrightarrow{\mathcal{I}} x$ if $\{n : |x_n - x| \geq \varepsilon\} \in \mathcal{I}$ for all $\varepsilon > 0$.
- $f_n \xrightarrow{\mathcal{I}} f$ (*$\mathcal{I}$-pointwise convergence*) if $f_n(x) \xrightarrow{\mathcal{I}} f(x)$ for all $x \in X$;
- $f_n \xrightarrow{\mathcal{I}-d} f$ (*$\mathcal{I}$-discrete convergence*) if $\{n : f_n(x) \neq f(x)\} \in \mathcal{I}$ for all $x \in X$;
- $f_n \xrightarrow{(\mathcal{I},\mathcal{J})-e} f$ (*$(\mathcal{I}, \mathcal{J})$-equal convergence*) if there exists a sequence of positive reals $(\varepsilon_n) \xrightarrow{\mathcal{J}} 0$ such that $\{n : |f_n(x) - f(x)| \geq \varepsilon_n\} \in \mathcal{I}$ for all $x \in X$.

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Classes of functions

Let X be a topological space. By $C(X)$, $B_\alpha(X)$ and $Baire\,(X)$ we denote the classes of all functions $f : X \to \mathbb{R}$ which are continuous, of Baire class α, and Baire measurable, respectively.

A function $f : X \to \mathbb{R}$ is:

- *quasi-continuous in* $x_0 \in X$ ($x \in C_q(f)$) if for every $\varepsilon > 0$ and open neighborhood U of x_0 there exists an open nonempty $V \subseteq U$ such that for all $x \in V$ we have $|f(x) - f(x_0)| < \varepsilon$;
- *quasi-continuous* ($f \in QC(X)$) if $C_q(f) = X$;
- *pointwise discontinuous* ($f \in PWD(X)$) if the set $C(f)$ of continuity points of f is dense in X;
- *in* $PWD_0(X)$ if the set $X \setminus C_q(f)$ is nowhere dense in X.

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Plan of the talk

	$\mathcal{I}$-pointwise convergence	$\mathcal{I}$-discrete convergence	$(\mathcal{I}, \mathcal{J})$-equal convergence
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Preliminaries

If $\mathcal{I}$ and $\mathcal{J}$ are ideals on X and Y, respectively, then we say that $\mathcal{J}$ *contains an isomorphic copy of* $\mathcal{I}$ ($\mathcal{I} \sqsubseteq \mathcal{J}$) if there is a bijection $f : X \to Y$ such that $f[A] \in \mathcal{J}$ for all $A \in \mathcal{I}$.

Ideals $\mathcal{I}$ and $\mathcal{J}$ on ω are *orthogonal* if there are $A \in \mathcal{I}$ and $B \in \mathcal{J}$ such that $A \cup B = \omega$.

Proposition (A.K. and M. Staniszewski)

If $\mathcal{I}$ and $\mathcal{J}$ are orthogonal ideals on ω, then $B_1^{(\mathcal{I},\mathcal{J})-e}(\mathcal{F}) = \mathbb{R}^X$ for any set X and any non-empty family of functions $\mathcal{F} \subseteq \mathbb{R}^X$.

Let $A \in \mathcal{P}(\omega)$ and $(A_n)_{n\in\omega} \subseteq \mathcal{P}(\omega)$. We denote:

- $\mathcal{I} \sqcup A = \{M \cup N : \ M \in \mathcal{I} \wedge N \subseteq A\}$;
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QC(X) + $\mathcal{I}$-pointwise convergence

Theorem (Z. Grande)

1. $B_1(QC(X)) = PWD(X)$ *for all metric Baire spaces* X.
2. $B_\alpha(QC(X)) = Baire(X)$ *for all* $\alpha > 1$ *and metric Baire spaces* X.

Theorem (P. Szuca and T. Natkaniec; A.K.)

Let $\mathcal{I}$ be a Borel ideal.

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Theorem (folklore)

1. $B_1^e(QC(X)) = PWD_0(X)$ *for all metric Baire spaces* X.
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Theorem (A.K. and M. Staniszewski)

Let $\mathcal{I}$ and $\mathcal{J}$ be non-orthogonal ideals on ω. Suppose that $\mathcal{I}$ is Borel.

1. *$(\mathcal{I},\mathcal{J})$ is of the first q-type if and only if $B_1^{(\mathcal{I},\mathcal{J})-e}(QC(X)) = PWD_0(X)$ for all metric Baire spaces X.*
2. *$(\mathcal{I},\mathcal{J})$ is of the second q-type if and only if $B_1^{(\mathcal{I},\mathcal{J})-e}(QC(X)) = PWD(X)$ for all metric Baire spaces X.*
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Definition

1. $(\mathcal{I}, \mathcal{J})$ *is of the first q-type if* $\mathcal{WR} \not\sqsubseteq \mathcal{I} \sqcup (A_n)_{n\in\omega}$ *for any sequence* $(A_n)_{n\in\omega} \subseteq \mathcal{J}$.
2. $(\mathcal{I}, \mathcal{J})$ *is of the second q-type if* $\mathcal{WR} \sqsubseteq \mathcal{I} \sqcup (A_n)_{n\in\omega}$ *for some sequence* $(A_n)_{n\in\omega} \subseteq \mathcal{J}$, *but* $\mathcal{WR} \not\sqsubseteq \mathcal{I} \sqcup A$ *for any* $A \in \mathcal{J}$.
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Example

1. $(\mathrm{Fin}, \mathrm{Fin} \otimes \emptyset)$ is of the first q-type.
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2. $(\mathcal{I}, \mathcal{J})$ *is of the second q-type if* $\mathcal{WR} \sqsubseteq \mathcal{I} \sqcup (A_n)_{n\in\omega}$ *for some sequence* $(A_n)_{n\in\omega} \subseteq \mathcal{J}$*, but* $\mathcal{WR} \not\sqsubseteq \mathcal{I} \sqcup A$ *for any* $A \in \mathcal{J}$.
3. $(\mathcal{I}, \mathcal{J})$ *is of the third q-type if* $\mathcal{WR} \sqsubseteq \mathcal{I} \sqcup A$ *for some* $A \in \mathcal{J}$.

Example

1. $(\mathrm{Fin}, \mathrm{Fin} \otimes \emptyset)$ is of the first q-type.
2. $(\emptyset \otimes \mathrm{Fin}, \mathrm{Fin} \otimes \emptyset)$ is of the second q-type.
3. $(\mathcal{WR}, \mathrm{Fin} \otimes \emptyset)$ is of the third q-type.

QC(X) + $(\mathcal{I}, \mathcal{J})$-equal convergence

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C(X) + $\mathcal{I}$-pointwise and $\mathcal{I}$-discrete convergence

Theorem (M. Katětov; M. Laczkovich and I. Recław; G. Debs and J. Saint Raymond; R. Filipów and P. Szuca)

Let $\mathcal{I}$ be a Borel ideal and $1 \leq \alpha < \omega_1$.

1. $Fin \otimes Fin \not\sqsubseteq \mathcal{I}$ *if and only if* $B_\alpha^{\mathcal{I}}(C(X)) = B_\alpha(X)$ *for all perfectly normal spaces* X.
2. $Fin \otimes Fin \sqsubseteq \mathcal{I}$ *if and only if* $B_\alpha^{\mathcal{I}}(C(X)) \supseteq B_{\alpha+1}(X)$ *for all perfectly normal spaces* X.

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Theorem (A.K. and M. Staniszewski)

Let $\mathcal{I}$ and $\mathcal{J}$ be non-orthogonal ideals on ω. Suppose that $\mathcal{I}$ is Borel and $1 \leq n < \omega$.

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Problem

Generalize the previous Theorem to all $1 \leq \alpha < \omega_1$.

Problem

Characterize $B_1^{(\mathcal{I},\mathcal{J})-e}(C(X))$ *for* $(\mathcal{I},\mathcal{J})$ *of the third c-type. Is it always equal to one of the classes* $B_\gamma^{(Fin,Fin)-e}(C(X))$ *or can it be equal to some* $B_\gamma(X)$?

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Thank you for your attention!